

LITERATURE CITED

1. L. Bighel, A. R. Collins, and R. C. Cross, "MHD 'switch-on' structure," *Phys. Lett.*, **A47**, No. 4 (1974).
2. R. Kh. Kurtmullaev, V. L. Masalov, K. I. Mekler, and V. N. Semenov, "Shock waves propagating along a magnetic field in a collision-free plasma," *Zh. Éksp. Teor. Fiz.*, **60**, No. 1 (1971).
3. K. Kuriki and M. Inutake, "Super-Alfvénic flow and collision-free shock wave in a plasma wind tunnel," *Phys. Fluids*, **17**, No. 1 (1974).
4. R. J. Bickerton, L. Lenamon, and R. V. W. Murphy, "The structure of hydromagnetic shock waves," *J. Plasma Phys.*, **5**, No. 2 (1971).

STATISTICAL - PHENOMENOLOGICAL APPROACH TO THE DESCRIPTION OF TURBULENT FLAMES

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INTRODUCTION

Development of a theory for the combustion of gaseous mixtures in a turbulent stream is usually associated with the modeling of the process on the basis of certain particular assumptions [1]. At the same time, in the hydrodynamics of nonreacting flows a method for the statistical averaging of the Navier - Stokes equations is being successfully developed. The resulting correlation moments of the second-order hydrodynamic quantities are not directly related to the average flow parameters; for these it is necessary to set up equations which are similar in structure to conservation laws. Although the higher-order moments which then appear again require the use of phenomenological hypotheses, the chain of two-moment equations constructed in this way does describe a number of qualitatively new effects. The fundamentals of this approach were laid down by A. N. Kolmogorov and have now been widely developed by Soviet and other scientists (see, for example, [2] and the references cited therein): Solutions have been constructed with a small number of empirical constants to give satisfactory qualitative description of a turbulent flow. A similar approach to this is used in [3] to describe a chemical reaction with a linear source of heat (concentration) and in [4] for thermal transport problems. The effect of nonlinear heat production with allowance for pulsations in temperature and concentration in the zeroth-order statement of the problem is studied in [5]; the application to a small-scale turbulent flame is considered in [6-9]. In these last papers, the flame-propagation equation is closed by using the phenomenology of the Prandtl displacement path; a method of averaging the nonlinear thermal production function is proposed. In [10] the equation for the turbulent energy balance is used to study the level of turbulence in a flame where the Prandtl hypothesis is also used for the temperature pulsations. In the present paper, the statistical-phenomenological method is extended to the problem of the turbulent combustion of mixed gases. The principal emphasis is on studying the effect of the chemical reaction on the thermal exchange in the flame and the reverse effect of the turbulence on the reaction rate and hence on the turbulent combustion rate. Approximate estimates of these effects are given.

§1. The presentation given below is based on the following simplifying assumptions: 1) the hydrodynamic field of the averaged $\langle u_i(x_i, t) \rangle$, $\langle p(x_i, t) \rangle$ and pulsating $u_i^{\prime}(x_i, t)$, $p^{\prime}(x_i, t)$ motions is known; 2) the medium is incompressible, $\rho = \text{const}$; 3) a single-stage exothermic reaction takes place in the flow in accordance with the equation $nA \rightarrow B$ at a rate $\Phi(T, \eta)$ and with calorific value $Q > 0$ (n is the order of the reaction; A is the initial substance; B are the reaction products; T is the temperature; and η is the relative concentration of the reaction products or the degree of combustion); 4) the molecular transport coefficients are independent of the flow parameters and they are subject to the equations $\nu = \kappa = D_{AB}$ (ν is the viscosity; κ is the thermal conductivity; and D is the diffusion); 5) we consider the usual slow combustion conditions when we can neglect the pressure gradient and the viscous dissipation heat in the flame in comparison with the heat developed by the chemical reaction; and 6) the back reaction of the flame on the hydrodynamics of the flow $\langle u_i \rangle$, u_i^{\prime} , p is not taken into ac-

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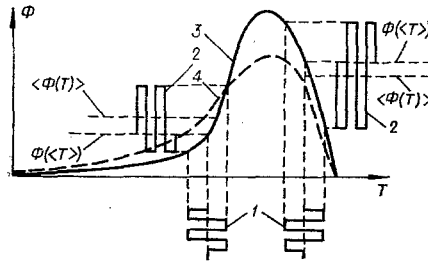


Fig. 1

TABLE 1

F	ω_f/ω_0 (5.4)	ω_f/ω_0 (4.8)-(4.11)
1	1,00	1,01
2	1,03	1,04
3	1,17	1,12
4	1,38	1,33

count. Estimates show [10] that this back effect is small, so that the intensity $b = \langle u_i u_j \rangle / 2$ and the macroscale L of the turbulence can serve in the first approximation as parameters of the problem (repeated indices imply summation).

With these assumptions we can use the following system of equations (in real units):

$$\partial u_j / \partial x_j = 0; \quad (1.1)$$

$$\partial u_i / \partial t + u_j \partial u_i / \partial x_j = -\rho^{-1} \partial p / \partial x_i + \nu \partial^2 u_i / \partial x_j \partial x_j; \quad (1.2)$$

$$\partial T / \partial t + u_j \partial T / \partial x_j = \nu \partial^2 T / \partial x_j \partial x_j + (Q/c\rho) \Phi(T, \eta); \quad (1.3)$$

$$\partial \eta / \partial t + u_j \partial \eta / \partial x_j = \nu \partial^2 \eta / \partial x_j \partial x_j + \Phi(T, \eta). \quad (1.4)$$

The expression

$$T = T_+ - (Q/c\rho)\eta \quad \text{or} \quad T = T_+ - (T_+ - T_-)\eta \quad (1.5)$$

is one particular solution of (1.3) and (1.4). The subscript "+" is used to denote quantities referring to the final products of the combustion and "-" is used for the original mixture. In combustion problems (1.5) also satisfies the boundary conditions and is therefore the first integral of (1.3) and (1.4). In what follows we consider only this case so that we can take the reaction-rate function Φ to depend on temperature alone.

We divide the actual quantities into average and pulsating parts ($T = \langle T \rangle + T'$, $u_i = \langle u_i \rangle + u_i'$, and so on) and we apply the Reynolds averaging operation to (1.3), so that with the help of (1.1) we get

$$\partial T / \partial t + \langle u_j \rangle \partial \langle T \rangle / \partial x_j = \nu \partial^2 \langle T \rangle / \partial x_j \partial x_j - \frac{\partial \langle u_j' T' \rangle}{\partial x_j} + (Q/c\rho) \langle \Phi(T) \rangle. \quad (1.6)$$

The application of the averaged thermal balance equation to the study of a turbulent flame is complicated by the presence of unknown quantities: the moment $\langle u_i' T' \rangle$ and the average heat-production function $\langle \Phi(T) \rangle$. For these we have to either use empirical connections (see, for example, [6-9]) with the average values or write out the balance equations which describe their variation.

The equation for the moment $\langle u_i' T' \rangle$ is

$$\begin{aligned} & \partial \langle u_i' T' \rangle / \partial t + \langle u_j \rangle \partial \langle u_i' T' \rangle / \partial x_j + \langle u_i' u_j' \rangle \partial \langle T \rangle / \partial x_j - \\ & - \nu \partial^2 \langle u_i' T' \rangle / \partial x_j \partial x_j + \langle u_j' T' \rangle \partial \langle u_i \rangle / \partial x_j + 2\nu \langle (\partial u_i' / \partial x_j) (\partial T' / \partial x_j) \rangle - \\ & - \rho^{-1} \langle p' \partial T' / \partial x_i \rangle + \partial \langle u_j' (u_i' T') \rangle / \partial x_j + \rho^{-1} \partial \langle p' T' \rangle / \partial x_i = (Q/c\rho) \langle u_i' \Phi \rangle. \end{aligned} \quad (1.7)$$

As in [4, 11], we close the system (1.7) by means of the following phenomenological hypotheses:

$$\rho^{-1} \langle p' \partial T' / \partial x_i \rangle = -c_1 \sqrt{b} \langle u_i' T' \rangle / L_q; \quad (1.8)$$

$$2\nu \langle (\partial u_i' / \partial x_j) (\partial T' / \partial x_j) \rangle = -c_2 \nu \langle u_i' T' \rangle / L_q^2; \quad (1.9)$$

$$\partial \langle u'_i (u'_i T') \rangle / \partial x_j = \partial (D_q \partial \langle u'_i T' \rangle / \partial x_j) / \partial x_j. \quad (1.10)$$

In these equations L_q represents the scale of the velocity-temperature correlation and it may differ from the hydrodynamic L_b ; b is the turbulence energy per unit mass and the c_i are coefficients.

The turbulent diffusion coefficient D_q cannot in general be identified with the diffusion coefficient of the pulsation kinetic energy $D_b \approx L_b \sqrt{b}$, since D_q can depend on the nature of the chemical heat-production process.

The term $-\rho^{-1} \partial \langle p' T' \rangle / \partial x_i$ is usually connected with the transport of energy in a nonuniform temperature field by the pressure pulsations. It is probably important near the walls and is omitted in the treatment which follows.

Final closure of the system of equations requires us to set up the balance equation for the temperature pulsation $\langle T'^2 \rangle$ [this quantity occurs in $\langle \Phi(T) \rangle$ and $\langle u'_i \Phi \rangle$]:

$$\begin{aligned} & \partial \langle T'^2 \rangle / \partial t + \langle u_j \rangle \partial \langle T'^2 \rangle / \partial x_j + 2 \langle u'_j T' \rangle \partial \langle T \rangle / \partial x_j - \\ & - \nu \partial^2 \langle T'^2 \rangle / \partial x_j \partial x_j + 2\nu \langle (\partial T' / \partial x_j) (\partial T' / \partial x_j) \rangle + \partial \langle u'_j T'^2 \rangle / \partial x_j = 2(Q/c_p) \langle T' \Phi \rangle. \end{aligned} \quad (1.11)$$

In a similar way we get

$$2\nu \langle (\partial T' / \partial x_j) (\partial T' / \partial x_j) \rangle = -c_3 \sqrt{b} \langle T'^2 \rangle / L_\sigma - c_4 \nu \langle T'^2 \rangle / L_\sigma^2; \quad (1.12)$$

$$\partial \langle u'_j T'^2 \rangle / \partial x_j = \partial (D_\sigma \partial \langle T'^2 \rangle / \partial x_j) / \partial x_j, \quad (1.13)$$

where L_σ is the temperature scale of the turbulence and D_σ is the turbulent diffusion coefficient of the temperature pulsations.

§2. In order to determine the quantities $\langle \Phi(T) \rangle$, $\langle T' \Phi \rangle$, and $\langle u'_i \Phi \rangle$, which appear on the right sides of (1.6), (1.7), and (1.11), we have to know the probability distribution functions of the temperature (velocity) pulsations.

Since no reliable data exist on the form of this function, we have to start from the simplest pulsation structure and assume that positive and negative pulsations T' with amplitude $\sqrt{\langle T'^2 \rangle}$ are equally likely. This means that the probability distribution function can be written as

$$P(T') = 2^{-1} [\delta(T' + \sqrt{\langle T'^2 \rangle}) + \delta(T' - \sqrt{\langle T'^2 \rangle})], \quad (2.1)$$

where $\delta(\dots)$ is the Dirac delta function.

The choice of this particular function makes our approach similar to the arithmetic averaging technique used in [6-9].

In fact, we have from (2.1)

$$\langle \Phi(T) \rangle = \int \Phi(\langle T \rangle + T') P(T') dT' = [\Phi(\langle T \rangle + \sqrt{\langle T'^2 \rangle}) + \Phi(\langle T \rangle - \sqrt{\langle T'^2 \rangle})] / 2. \quad (2.2)$$

In a similar way, we get for the moment $\langle T' \Phi \rangle$,

$$\langle T' \Phi \rangle = \int T' \Phi(\langle T \rangle + T') P(T') dT' = \sqrt{\langle T'^2 \rangle} [\Phi(\langle T \rangle + \sqrt{\langle T'^2 \rangle}) - \Phi(\langle T \rangle - \sqrt{\langle T'^2 \rangle})] / 2. \quad (2.3)$$

We can calculate the moment $\langle u'_i \Phi \rangle$ from the equation

$$\langle u'_i \Phi \rangle = \int u'_i T' (\Phi(\langle T \rangle + T') / T') P_1(T') P_2(u'_i T') dT' d(u'_i T'),$$

where the functions P_1 and P_2 are chosen to depend on the moments which contain the temperature pulsations, so that we finally get arithmetic averaging, and in the limit when $\sqrt{\langle T'^2 \rangle} \ll \langle T \rangle$,

$$\langle u'_i \Phi \rangle = \langle u'_i T' \rangle [\Phi(\langle T \rangle - \sqrt{\langle T'^2 \rangle}) - \Phi(\langle T \rangle + \sqrt{\langle T'^2 \rangle})] / 2\sqrt{\langle T'^2 \rangle}. \quad (2.4)$$

In particular, for small $\sqrt{\langle T'^2 \rangle} \ll \langle T \rangle$ we get the following expansions from (2.2)-(2.4):

$$\langle \Phi(T) \rangle = \Phi(\langle T \rangle) + \langle T'^2 \rangle d^2 \Phi(\langle T \rangle) / 2d\langle T \rangle^2 + O(T'^4); \quad (2.5)$$

$$\langle T' \Phi \rangle = \langle T'^2 \rangle d\Phi(\langle T \rangle) / d\langle T \rangle + O(T'^4); \quad (2.6)$$

$$\langle u'_i \Phi \rangle = (d\Phi(\langle T \rangle) / d\langle T \rangle) \langle u'_i T' \rangle + O(\langle u'_i T'^3 \rangle). \quad (2.7)$$

We might note that these same approximations [to the accuracy in (2.5)-(2.7)] also follow directly from the expansions of the functions $\Phi(\langle T \rangle + T')$, $T'\Phi(\langle T \rangle + T')$, $u_1'\Phi(\langle T \rangle + T')$ into series in powers of T' and subsequent Reynolds averaging. This result is valid for any probability distribution function.

The deviation of the average reaction rate from the value at the mean temperature is governed according to (2.5), mainly by the curvature of the function $\Phi(\langle T \rangle)$. This fact, which was noted in [8], is illustrated in Fig. 1 with a typical $\Phi(T)$ relationship for a first-order reaction (curve 1 is the temperature pulse; 2 is the pulsation of the reaction rate; 3 is the heat production in the laminar case; and 4 is the averaged heat-production curve). When the reaction is perturbed in the high-temperature region ($T_+ - T_- \ll T_+$) $d^2\Phi/d\langle T \rangle^2 < 0$, the average reaction rate becomes smaller than the rate at the mean temperature. In the temperature region where $d^2\Phi/d\langle T \rangle^2 > 0$, the opposite effect occurs. Of course, continuous variations of temperature distort both the $\Phi(\langle T \rangle)$ profile (in comparison with the laminar case) and the flame structure. The energy relations (1.7), (1.11) and (2.6), (2.7) show that the chemical reaction generates temperature (and therefore $\langle u_1'T' \rangle$) pulsations in the temperature interval where the heat production increases and absorbs them in the final stage of the reaction where as a result of the burning of the original components the reaction rate decreases with temperature. Quantitative estimates of this effect are given below.

Equations (1.6), (1.7), and (1.11) together with the closure expressions (1.8)-(1.10), (1.12), (1.13), and (2.2)-(2.4), fully describe the chemical reaction process in a turbulent flow of mixed gases. The statistical moments which contain only the velocity $\langle u_1'u_1' \rangle$ and the turbulence scales L_q and L_σ are assumed to be known functions of the coordinates and time. It is possible to set up equations for these quantities; a method for getting the equation for the scale is suggested, for example, in [12].

We do not elaborate here on the concept of the quantity L , although it is fairly clear that for the flow center in a tube, L is proportional to the outer scale of the turbulence, i.e., to the diameter of the tube. In the study of boundary-layer flows the significance of L might change [13].

It is important to note that the coefficients c_1 , c_2 , c_3 , and c_4 which occur in the factors in front of L_q and L_σ are determined by experiments and this to some extent compensates for the fact that the exact values of L_q and L_σ are unknown.

When there is no similarity between the concentration and the temperature it becomes necessary to set up additional equations for the statistical moments $\langle \eta'^2 \rangle$, $\langle \eta'T' \rangle$, $\langle u_1'\eta' \rangle$.

§3. Considering the simplest case of a stationary flow where $\langle u_1 \rangle = u = \text{const}$, $\langle u_2 \rangle = \langle u_3 \rangle = 0$, $\langle T \rangle = \langle T(x) \rangle$, we arrive at equations which describe a one-dimensional model of a turbulent flame (the analog of a normal flame). This case occurs in tubes (or beyond vortex plates) in the flow core where the walls have no effect and in certain other situations of free turbulent flow.

In a coordinate system linked to the combustion surface, the quantity $\langle u_1 \rangle$ is simply the velocity of the turbulent propagation of the flame u_t . In the more general case where the flame is carried by the flow, the value of $\langle u_1 \rangle = u + u_t$, where u is the constant "drift" velocity. However, even here the average velocity u can be eliminated by an appropriate transformation of coordinates. It is an important fact that in this approach no restrictions are placed on the magnitude of the flow pulsation energy in comparison with the turbulent combustion rate; i.e., \sqrt{b} can be either greater or less than u_t and the level of the turbulence remains small compared to the main flow, $\sqrt{b} \ll \langle u_1 \rangle$.

These assumptions lead to the following system of equations for the turbulent flame propagation:

$$d(xdT/\partial x - q)/\partial x - u_t dT/dx + (Q/c\rho)\Phi_*(T, |T'|) = 0; \quad (3.1)$$

$$D_q d^2q/dx^2 - u_t dq/dx = |u_t|^2 |dT/dx + c_1 \sqrt{b}q/L_q - (Q/c\rho)\Phi_{**}(T, |T'|)q; \quad (3.2)$$

\downarrow
 $O(L^2/\delta^2)$

\downarrow
 $O(L/\delta)$

\downarrow
 $O(1)$

\downarrow
 $O(1)$

\downarrow
 $O(\tau_{1q}/\tau_x)$

$$D_\sigma d^2\langle T'^2 \rangle/dx^2 - u_t d\langle T'^2 \rangle/dx = 2qdT/dx + c_3 \sqrt{b} |T'|^2/L_\sigma - (2Q/c\rho)\Phi_{**}(T, |T'|) |T'|^2, \quad (3.3)$$

\downarrow
 $O(L^2/\delta^2)$

\downarrow
 $O(L/\delta)$

\downarrow
 $O(1)$

\downarrow
 $O(1)$

\downarrow
 $O(\tau_{1\sigma}/\tau_x)$

where the order of each term is shown beneath that term.

In order to simplify the writing we omit the averaging sign and introduce the following new notation:

$$\begin{aligned}\sqrt{\langle u'^2 \rangle} &= |u'|, \quad q = \langle u'T' \rangle, \quad |T'| = \sqrt{\langle T'^2 \rangle}, \\ 2\Phi_* &= \Phi(T + |T'|) + \Phi(T - |T'|), \\ 2\Phi_{**} &= \Phi(T + |T'|) - \Phi(T - |T'|);\end{aligned}$$

$\tau_{1q} = L_q/c_1\sqrt{\bar{b}}$, $\tau_{1\sigma} = L_\sigma/c_3\sqrt{\bar{b}}$ are the characteristic hydrodynamic times (the time for the displacement of large moles); τ_X is the characteristic time of the chemical reaction. Equations (3.2) and (3.3) are valid only for the flow core, where

$$\kappa/D_q \ll 1, \quad \kappa/D_\sigma \ll 1, \quad c_2\kappa/c_1L_q\sqrt{\bar{b}} \ll 1, \quad c_4\kappa/c_3L_\sigma\sqrt{\bar{b}} \ll 1. \quad (3.4)$$

The quantity κ in (3.1) is retained for the possible limiting transition of the combustion in a laminar flow.

In the intermediate layer of the boundary flow (or in the viscous subflow), the inequalities (3.4) are not satisfied and additional factors appear in (3.2) and (3.3): in place of D_q we have to write $D_q(1 + \kappa/D_q)$, in place of c_1 we have $c_1(1 + c_2\kappa/c_1L_q\sqrt{\bar{b}})$, in place of D_σ we have $D_\sigma(1 + \kappa/D_\sigma)$ and in place of c_3 we write $c_3(1 + c_4\kappa/c_3L_\sigma\sqrt{\bar{b}})$.

The contributions of each term to the energy balance of the T' and $u'T'$ pulsations are characterized by the relative sizes attached to these terms in (3.2) and (3.3). A significant change in the various quantities occurs at a distance on the order of the width of the turbulent-flame front δ , and it is moreover assumed that $D_b \approx L\sqrt{\bar{b}}$, $D_b \approx D_q \approx D_\sigma$, $u_t \approx u'$ [this choice of u_t gives an upper estimate of the second terms on the left sides of (3.2) and (3.3), $q \approx L\sqrt{\bar{b}}dT/dx \approx L\sqrt{\bar{b}}T_+/\delta$, $L \approx L_q \approx L_\delta$].

§ 4. We consider the case of turbulent combustion when $L \ll \delta$. For small-scale turbulent combustion, δ can be considerably greater than the corresponding width of a laminar flame δ_0 : The extension of the turbulent flame is caused by the increased heat transport and an additional extension (first-order reaction) by the shift in the maximum of the heat production toward lower temperatures [7]. The same is true for the characteristic reaction time τ_X : In a turbulent flame τ_X can greatly exceed τ_+ —the reaction time at the maximum temperature and the initial concentration.

It can be seen that for the present problem the transport of pulsation energy by turbulent diffusion and convection is unimportant, so that an energy balance is set up between the creation of fluctuations at the gradient of the average temperature, the dissipation, and the generation (absorption) by the chemical reactions.

Dropping the unimportant terms in (3.2) and (3.3), we have

$$|u'|^2dT/dx - (Q/c_p)q\Phi_{**}(T, |T'|)/|T'| + c_1\sqrt{\bar{b}}/L_q = 0; \quad (4.1)$$

$$qdT/dx - (Q/c_p)|T'|^2\Phi_{**}(T, |T'|) + (c_3\sqrt{\bar{b}}/2L_\sigma)|T'|^2 = 0. \quad (4.2)$$

If the scales and coefficients of the turbulence are related by the equation[†]

$$2c_1L_\sigma = c_3L_q, \quad (4.3)$$

then a change in $|T'|$ is similar to a change in q :

$$q = -|u'| |T'|. \quad (4.4)$$

The minus sign is chosen because the turbulent thermal conductivity is positive.

The flame propagation is described by Eqs. (3.1), (4.2), and (4.4); after some simple algebra these can be transformed to

$$d|T'|/dT = u_t/|u'| - \Phi_*(T, |T'|)/[|u'|c_p|T'|/LQ - \Phi_{**}(T, |T'|)]; \quad (4.5)$$

$$|T'|(T_+) = 0, \quad |T'|(T_-) = 0; \quad (4.6)$$

$$dT/dx = |T'|/L - (Q/c_p|u'|)\Phi_{**}(T, |T'|). \quad (4.7)$$

The solution of the boundary-value problem (4.5), (4.6) gives the turbulent combustion rate u_t and (4.7) gives the temperature distribution profile.

In order to compare our results with those in [7-9], we reformulate (4.5)-(4.7) in dimensionless variables:

$$dp/du = \Phi_*(u, p)/[p + F_0\Phi_{**}(u, p)] - \omega_t; \quad (4.8)$$

[†]The treatment of the more general case, where (4.3) is not satisfied, does not present any fundamental difficulties and leads only to a more complicated final result.

$$p_{(0)} = p_{(1)} = 0; \quad (4.9)$$

$$\xi = - \int_0^u \frac{du}{p + F_0 \Phi_{**}(u, p)}; \quad (4.10)$$

$$2\Phi_{**} = (u + F_0 p)^n \exp[-\theta_0(u + F_0 p)/(1 - \sigma(u + F_0 p))] \pm (u - F_0 p)^n \exp[-\theta_0(u - F_0 p)/(1 - \sigma(u - F_0 p))] \quad (4.11)$$

for $0 \leq u \leq \varepsilon$; when $\varepsilon < u < 1$, $\Phi_{**} = 0$ (the "+" sign is chosen for Φ_* and the "-" sign for Φ_{**}).

The relation between the dimensioned and dimensionless quantities is

$$\begin{aligned} u &= (T_+ - T)/(T_+ - T_-), \quad p = |T'|/F_0(T_+ - T_-), \\ \omega_t &= u_t \sqrt{\tau_+/L u'}, \\ \tau_+ &= \rho^{1-n} z^{-1} \exp(E/RT_+), \quad x_0 = \sqrt{L|u'| \tau_+}, \quad \xi = x/x_0, \\ F_0 &= L/x_0 = \sqrt{\tau_1/\tau_+}, \quad \tau_1 = L/|u'|, \\ \sigma &= 1 - T_-/T_+, \quad \theta_0 = E(T_+ - T_-)/RT_+^2. \end{aligned}$$

The scale L is proportional to L_q ,

$$L = L_q/c_5 c_5,$$

where the coefficient c_5 allows for a possible hydrodynamic anisotropy in the flow, $b = c_5 |u'|^2$. For an isotropic flow $c_5 \approx \sqrt{3}/2$. The average value $c_1 \approx 2.5$ has been determined for a nonreacting flow [13]. In order to make the comparison with the earlier results we have to remember that $F_0 = F/\sqrt{c_1 c_5} \approx F/\sqrt{3}$.

§5. We consider the limiting case $|T'| \ll T$, where we can come to some definite qualitative conclusions. Using the expansion (2.6) and Eq. (4.3) we get the gradient representations

$$q = \langle u' T' \rangle = -L|u'| [1 - (Q\tau_1/c\rho)d\Phi/dT]^{-1} dT/dx; \quad (5.1)$$

$$|T'| = L [1 - (Q\tau_1/c\rho)d\Phi/dT]^{-1} |dT/dx|. \quad (5.2)$$

In the absence of any chemical reactions (or when $\tau_1/\tau_x \ll 1$) (5.1) and (5.2) give the classical result of displacement path theory. The presence of chemical heat production increases the turbulent thermal conductivity κ_t in that region of the flame where $d\Phi/dT > 0$ and decreases it in the final stages of the reaction when the quantity $d\Phi/dT$ (see Fig. 1) has changed sign. The quantity Φ_{**} in (4.8) reflects the role of the chemical reaction in the thermal exchange inside the flame.

An approximate estimate of this effect on the turbulent combustion rate can be obtained from (4.8) by neglecting the influence of the temperature pulsations on Φ_{**} . After integrating (4.8) by the Zel'dovich method we have for a first-order reaction that

$$\omega_T^2 \approx 2 \int_0^1 \frac{u e^{-\theta_0 u}}{1 + 3^{-1} F^2 (1 - \theta_0 u) e^{-\theta_0 u}} du. \quad (5.3)$$

Making a power expansion of (5.3) and carrying out the integration, we get

$$\omega_t/\omega_0 \approx \sqrt{1 + F^4/3^5}, \quad (5.4)$$

where $\omega_0 = \sqrt{2}/\theta_0$ is the laminar combustion rate. Equation (5.4) is valid when

$$F < e\sqrt{3} \approx 4.7, \quad \tau_1/\tau_+ \leq 7.29. \quad (5.5)$$

The limit (5.5) represents an estimate which is valid for $|T'| \ll T$. In general, it cannot be attained: As L increases, there is a rise in the time for complete reaction resulting from the distortion of the heat-production pulsations. Table 1 gives the results of calculations from (5.4) compared with those obtained from (4.8)-(4.11). In using (4.11) we have neglected the effect of temperature pulsations on Φ_* and have represented Φ_{**} in the expanded form as valid for $F_0 p \ll 1$, as in (2.6) and (2.7).

It is necessary to distinguish between small-scale flames in free and boundary-layer turbulent flows (in the latter case it is necessary to allow for the dependence of L on the distance from the wall, the variability of the $|u'|$ pulsations, and so on). Moreover, in the study of the turbulent combustion in the boundary-layer regions (which in practice means in the intermediate region of the boundary layer [14, 15]) we cannot neglect the quantity $\kappa/L\sqrt{b}$ in comparison with unity.

LITERATURE CITED

1. K. I. Shchelkin and Ya. K. Troshin, *The Gasdynamics of Combustion* [in Russian], Izd. Akad. Nauk SSSR, Moscow (1963).
2. A. S. Monin and A. M. Yaglom, *Statistical Hydromechanics* [in Russian], Vol. 1, Nauka, Moscow (1965).
3. F. A. Williams, "An approach to turbulent flame theory," *J. Fluid Mech.*, **40**, No. 2, 401 (1970).
4. B. A. Kolovandin, "The calculation of principal thermal exchange properties in turbulent flows with a drift motion," in: *Heat and Mass Transport* [in Russian], Vol. 1, Énergiya, Moscow (1968).
5. L. A. Vulis, "The rate of turbulent combustion," *Fiz. Goreniya Vzryva*, No. 1 (1972).
6. V. N. Vilyunov, "Certain problems in the theory of a turbulent flame," in: *Second Conference on Mathematics and Mechanics* [in Russian], Vol. 2, Izd. Tomsk. Univ., Tomsk (1972).
7. V. S. Baushev and V. N. Vilyunov, "Rate of propagation and limits of existence of a turbulent flame," *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 3 (1972).
8. V. S. Baushev and V. N. Vilyunov, "Mathematical theory of the stationary propagation velocity of a small-scale turbulent flame," *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 4 (1973).
9. V. N. Vilyunov and I. G. Dik, "Stationary combustion conditions in a small-scale turbulent flow," *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 3 (1973).
10. V. B. Librovich and V. I. Lisitsyn, "Energy balance of the turbulent pulsations in reacting flows," in: *Fourth All-Union Symposium on Burning and Explosion* [in Russian], OIKhF, Akad. Nauk SSSR, Chernogolovka (1974).
11. A. V. Lykov, *Thermo-Mass Exchange* [in Russian], Énergiya, Moscow (1972).
12. G. S. Glushko, "Differential equation for the scale of turbulence and calculation of a turbulent boundary layer at a plate," in: *Turbulent Flows* [in Russian], Nauka, Moscow (1970).
13. N. I. Akatnov, "Linear scales of turbulence in semiempirical theory," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 3 (1974).
14. V. N. Vilyunov, "The theory of the erosion burning of powders," *Dokl. Akad. Nauk SSSR*, **136**, No. 2 (1961).
15. V. N. Vilyunov and A. A. Dvoryashin, "Burning of a powder in a gas stream," *Fiz. Goreniya Vzryva*, No. 1 (1971).

CERTAIN FORMULATIONS OF THREE-DIMENSIONAL OPTIMIZATION PROBLEMS IN HYPERSONIC AERODYNAMICS

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The fundamental configuration of a prospective hypersonic aircraft in which the active balance of forces is created by a direct-flow air-breathing jet engine in liquid hydrogen with supersonic combustion is dictated by its specific functioning conditions. Thus, in order to ensure the intake of air from the atmosphere during flight in a rarefied medium the air-intake system should have a reasonably wide capture area, which will in fact differ very little from the middle cross section of the whole aircraft. The nozzle (second element in the engine system) should also have large dimensions. These engine elements should make a specific contribution to the aerodynamics of the aircraft as a whole; they are characterized by large areas immersed in the flow, on which the function of carrying surfaces will to a certain extent be imposed. Hence we have the necessity of asymmetry in the configurations of such surfaces and the associated essentially three-dimensional character of the perturbed flow.

Let us consider the following presentation of the fundamental problem: in a three-dimensional space we have two specified arbitrary closed contours l_1 and l_2 (Fig. 1); the isobars of the unknown flow are based on these contours, the pressures on the latter being specified as p_1 and p_2 , respectively. It is required to find the stream surface passing through both contours and optimizing a certain integrated force characteristic of the unknown surface. The problem is made specific by giving the functional of the mechanical (force) action.

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